

THE MAGIC CUBE SOLUTION

Introduction

The 'Magic' cube consists of 26 small cubes, each cleverly shaped so that they fit together to form a 3x3x3 larger cube. Any one of the six faces may be rotated in turn, making it possible to alter the relative positions of the small cubes.

Basic Observations

The small cubes, or PIECES, are of three distinct types.

1. Those with one coloured face, at the CENTRE of each face.
2. Those with two coloured faces, the EDGES of the cube.
3. Those with three coloured faces, the CORNERS of the cube.

The colour of a face is set by the colour of the centre piece, since this piece can only be rotated, never moved away.

The nine pieces which go together to form a face of the cube, in fact constitute a complete SLICE of the cube. These pieces are only in their correct positions if all their faces match the adjacent centres.

Handling Hints

Most beginners confuse themselves by holding the cube in a different way after turning a face. It is very important not to lose sight of the orientation of the cube while making a move, the best method being to remember the colours of the centres of the top and front faces.

Notation

Since the colours of the faces vary from one batch of cubes to another, it is necessary to have a way of describing moves without referring to colours. This has the advantage of making the moves independent of the orientation of the cube, providing it is not changed during the move.

The notation chosen is based on the physical position of the faces at the start of a move, and consists of the initial letters of the words Left, Right, Front, Back, Up, and Down.

Figure 1 shows the cube with three of the faces appropriately lettered. We can now refer to individual pieces by the letters on their faces, eg. the upper front edge will be 'UF', while the corner to the right of it will be 'URF'.

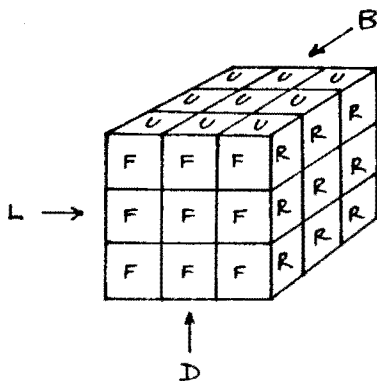


FIGURE 1.

A single 90 degree clockwise turn of a face will be denoted by the face letter, eg. 'F' means move the front face clockwise one quarter turn. 180 degrees is denoted by 'FF' or F^2 , and 270 degrees by F^{-1} (90 degrees anticlockwise).

These moves are always written assuming the face is being viewed on its coloured side, so we must be careful not to turn the D, L, and B faces the wrong way.

Let us examine a move, say F^2 , in detail. Try it a couple of times, watching what happens to the piece from FD. We say that FD has moved to FU, which means that the F face of FD becomes the F face of FU, and the D face of FD has become the U face of FU. So we have not only described the position the piece has moved to, but the orientation of it in its new position. You might like to check the eight other pieces which have changed position:

FU to FD, FL to FR, FR to FL, FUR to FDL, FDL to FUR, FUL to FDR, and FDR to FUL

If we had wanted to move FD to UF, then the sequence of moves $D.R.F^{-1}.R^{-1}$. would have been required.

Outline Process

The method to be described of restoring the cube to its start position, consists of the following stages.

1. Choose a colour for the 'U' face.
2. Put the four top edges correctly in place.
3. Put the four top corners correctly in place.
4. Put the four middle slice edges correctly in place.
5. Turn the cube over, so that the 'D' face is now the 'U' face.
6. Put the top edges in place.
7. Put the top corners in place.
8. Flip the top edges as required, to correctly align the colours.
9. Twist the top corners into their correct orientation.

Stage 1.

The colour you choose to have on the top face initially is immaterial, although you will find it easier if you always use the same colour.

Stage 2.

This is the easiest part of the process, because there are no pieces that can be disturbed. If any of the pieces are accidentally correct at this stage, ignore them.

Let us start by putting the FU piece in place. First note the colour of the F face, then find the edge piece which has the F and U colours, and move it down into the bottom (D) slice. This can be done by turning the slice containing the piece by either 90 or 180 degrees, one way or the other. Ensure you are still holding the cube in its original orientation, with the U and F face centre colours correct.

Now, by turning the D slice, move your piece into position FD, i.e. bring it to the front of the bottom slice by rotating the bottom slice until it appears. The piece will be in one of two orientations, since its F colour may be located in either the F or D face.

If the F colour is in the F face, then the move F^2 will position it correctly, i.e. move FD to FU. If the F colour is located in the D face, then you must move DF to FU by the sequence $D.R.F^{-1}.R^{-1}$.

This process can be repeated for each of the remaining three top edge pieces by holding the cube so that another colour is at the front. Note however, that you may now be disturbing an already correct top edge piece when you first move your next piece into the bottom slice. To avoid this, the first turn must be reversed after the second, eg. say the required piece is at RB, then the moves required for RB to FD are $R.D^{-1}.R^{-1}$.

Stage 3.

This stage is concerned with completing the top slice of the cube, by placing the four corners correctly, without of course disturbing the four edge pieces.

Choose a corner that you are going to work on, and find where it is at the moment. For simplicity, try to choose one which is already in the bottom slice. Then hold the cube with its U face upwards and the destination for the corner in position FUR,

and turn the bottom slice until the required piece is positioned at FRD. It has three possible orientations, each one requiring a different sequence of moves to place it correctly.

Remembering how we specify the effect of a move by the order of the letters in the piece names, choose the move you require from the three given.

- a. FRD to URF = $D^{-1}.R^{-1}.D.R.$
- b. FRD to FUR = $D.F.D^{-1}.F^{-1}.$
- c. FRD to RFU = $R^{-1}.D^2.R.D^2.F.D^{-1}.F^{-1}.$

Once again, repeat this process for the other three corners of the top slice. If you have to get a piece out of the top slice first, then the same type of move can be used, eg. to move FUR to DLF, for instance, perform $R^{-1}.D^{-1}.R.$

Stage 4.

The middle slice only requires its four corners to be inserted, since its edge pieces are in fact the centres of the F, L, B, and R faces, and are therefore already correct.

There are two moves, one the complement of the other, which are sufficient to complete the middle slice, the choice being governed by the orientation of the piece being inserted. If the required piece is located (incorrectly) in the middle slice, then one of these moves should be used to transfer it to the bottom slice, it does not matter which.

- a. LD to RF = $R^{-1}.D^{-1}.R.D^2.R^{-1}.D^2.R.D^2.R^{-1}.D.R.$
- b. RD to LF = $L.D.L^{-1}.D^2.L.D^2.L^{-1}.D^2.L.D^{-1}.L^{-1}.$

You will notice that the piece is moved from a bottom edge on one side to a front edge on the opposite side, so the choice of front face and move 'a' or 'b' is dependent on the vertical colour of the piece in the bottom slice. In other words, turn the bottom slice until the vertical colour of the piece is on the opposite side of the cube to the the face of the same colour, then hold the cube so that the destination is in the front slice while the piece is in either the left or right slice.

Stage 5.

Now we only have one slice left to do. Of course, it is the most difficult one.

At present, it is the bottom slice, but at this point we turn the cube over so that the incorrect slice becomes the top slice. In the remaining stages, the U face is the one which up to now has been the D face.

Stage 6.

The first objective is to get the pieces into their respective positions, without bothering about whether they are the right way up or not, and starting with the two faced edge pieces.

Since all the pieces are now in the correct horizontal slice, by turning the U slice it must be possible to put at least one edge piece in its right place. Having done this, examine the other three edges; you will find that they are either all correct, all incorrect, or one other is correct and the other two reversed.

If they are all correct, go on thankfully to stage 7. If two opposite edges are correct, then hold the cube so that one of the correct edges is at the front, and perform $R.U.R^{-1}.U.R.U^2.R^{-1}$ to make two adjacent edges correct. Then

If two adjacent edges are correct, turn the U slice so that only one of the four edges is correct. Then

With only one correct edge, hold the cube so the correct edge is at the front, and determine whether a clockwise or anticlockwise cycle of the other edges is needed to put them in their correct places.

Clockwise: UR to UL , UL to UB , UB to $UR = R.U^2.R^{-1}.U^{-1}.R.U^{-1}.R^{-1}$.

Anticlock: UL to UR , UR to UB , UB to $UL = R.U.R^{-1}.U.R.U^2.R^{-1}$.

Stage 7.

The four top corners can now be positioned, once again without regard to their orientation. To do this, we use a sequence which swaps two adjacent corners without otherwise altering the top slice. However, with this move we introduce a new concept, because pieces are disturbed in the two lower slices. Now obviously this can be corrected if we perform the disruptive sequence in exactly the reverse order, but what is the use of that?

Well, it is here that fortune smiles on us, because you will always find that these moves are required in pairs, and if the top slice is turned to another position in between the move and its converse, then two useful actions are completed by the time that the lower slices are restored. Needless to say, if you forget which colour the F face should be in between the moves, then complete chaos will result. This principle is also used in stages 8 and 9.

The move we require at this stage swaps the corners of the front face.

FLU to URF, URF to FLU = $F.R.F^{-1}.R^{-1}.F.R.F^{-1}.R^{-1}.F.R.F^{-1}.R^{-1}$.

We can write this move more simply $(F.R.F^{-1}.R^{-1})^3$

Having exchanged two of the corners using this move, and keeping the F face the same, turn the U slice until the other two corners to be swapped are in the F slice, and repeat the sequence. (Yes, in this case only, the move and its converse do the same thing).

The choice of corners to be exchanged must be worked out first, of course. They may be two distinct pairs of corners, or they may have a common piece if one corner is already correct, or two whole pairs of moves may be required to swap opposite corners.

Stage 8.

Now all the pieces are in their respective places, we can see how many need to be turned over, and unless you have had the cube to bits and assembled it jumbled up, an even number of edges will need to be flipped, i.e. 0, 2, or 4.

We will use a sequence which flips the UF piece, and is perhaps the most disruptive move we will encounter, so great care must be taken to keep the F face constant. The converse move will restore the lower slices and flip another piece put in position UF for that purpose, by turning the U slice between moves.

a. UF to FU = $F.U.D^{-1}.L^2.U^2.D^2.R.U$.

b. Converse = $U^{-1}.R^{-1}.D^2.U^2.L^2.D.U^{-1}.F^{-1}$.

Stage 9.

Nearly finished now, we should only have some corners to twist.

Since the corners each have three possible orientations, we must examine them carefully to see exactly what is needed to put them right. I find it best to imagine that we are going to twist a corner either clockwise or anticlockwise, as viewed from the outside corner. Using our notation, and examining the FUR piece, then a clockwise twist will move FUR to URF, and anticlockwise FUR to RFU.

The sequence we are going to use twists FUR clockwise, and its converse twists the position anticlockwise. You will find that your cube requires a number of these pairs of twists, to correct either 2, 3, or 4 corners. If you do have three corners to twist, then they will all seem to require one twist in the same direction, but you will choose one to receive two twists in the opposite direction, which is of course the same as one in the right direction.

a. Clockwise: FUR to URF = $R^{-1}.D.R.F.D.F^{-1}$.

b. Converse : FUR to RFU = $F.D^{-1}.F^{-1}.R^{-1}.D^{-1}.R$.

Congratulations ! If you have got to this point without making any mistakes, then you now should have a cube with a single colour on each face. Apart from astounding your friends by learning the sequences off by heart, you may like to play with a few symmetrical patterns given below.

Patterns.

Starting with the plain faces, the cube can be moved easily into some interesting patterns. To restore it, perform the moves exactly in reverse, eg. reading backwards and turning the opposite way.

Spot Face = $R.L^{-1}.U.D^{-1}.F^{-1}.B.R.L^{-1}$.

Cross Face = $R^2.L^2.U^2.D^2.F^2.B^2$.

2L's Face = $R.L.U.D.F^{-1}.B^{-1}.R.L$.

Diagonals = $(F.B.R.L.)^3$